From Nakayama-san's memo

Weak focus: Support documentation

Equations of motion in the field index-n are:

$$\frac{d^2x}{ds^2}+\frac{1}{\rho}(1-n)x=\frac{1}{\rho}\frac{\Delta p}{p}$$

$$\frac{d^2y}{ds^2}+\frac{1}{\rho^2}ny=0$$

$$n=-\frac{\rho}{B_0}\frac{\partial B_y}{\partial x}$$

B_0=3T,
p=300MeV/*c*
ρ=33.3cm

(Δp/p is momentum error)

Results of these equation are:

$$\begin{pmatrix} x(s) \\ x'(s) \\ \frac{\Delta p}{p}(s) \end{pmatrix} = \begin{pmatrix} \cos\frac{\sqrt{1-n}}{\rho}s & \frac{\rho}{\sqrt{1-n}}\sin\frac{\sqrt{1-n}}{\rho}s & \frac{\rho}{1-n}\left(1-\cos\frac{\sqrt{1-n}}{\rho}s\right) \\ -\frac{\sqrt{1-n}}{\rho}\sin\frac{\sqrt{1-n}}{\rho}s & \cos\frac{\sqrt{1-n}}{\rho}s & \frac{\rho}{\sqrt{1-n}}\sin\frac{\sqrt{1-n}}{\rho}s \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ \left.\frac{\Delta p}{p}\right|_0 \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} \cos\frac{\sqrt{1-n}}{\rho}s & \frac{\rho}{\sqrt{n}}\sin\frac{\sqrt{n}}{\rho}s \\ -\frac{\rho}{\sqrt{n}}\sin\frac{\sqrt{n}}{\rho}s & \cos\frac{\sqrt{1-n}}{\rho}s \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

Cont.

One-revolution transfer matrix is:

$$M_H = \begin{pmatrix} \cos 2\pi\sqrt{1-n} & \frac{\rho}{\sqrt{1-n}}\sin 2\pi\sqrt{1-n} \\ -\frac{\sqrt{1-n}}{\rho}\sin 2\pi\sqrt{1-n} & \cos 2\pi\sqrt{1-n} \end{pmatrix}$$

$$M_V = \begin{pmatrix} \cos 2\pi n & \frac{\rho}{\sqrt{n}}\sin 2\pi n \\ -\frac{\sqrt{n}}{\rho}\sin 2\pi n & \cos 2\pi n \end{pmatrix}$$

Matrix at s=0=2π is expressed by using twiss parameters α(s), β(s), γ(s) and phase advance μ(s).

$$M = \begin{pmatrix} \cos\mu(0)+\alpha(0)\sin\mu(0) & \beta(0)\sin\mu(0) \\ -\gamma(0)\sin\mu(0) & \cos\mu(0)-\alpha(0)\sin\mu(0) \end{pmatrix}$$

$$\mu_H(0) = 2\pi\sqrt{1-n},\ \nu_H(0) = \frac{\mu_H(0)}{2\pi} = \sqrt{n-1}$$

$$\beta_H(0) = \frac{\rho}{\sqrt{1-n}},\ \alpha_H(0) = 0,\ \gamma_H(0) = \frac{1+\alpha^2}{\beta} = \frac{\sqrt{1-n}}{\rho}$$

$$\mu_V(0) = 2\pi\sqrt{n},\ \nu_H(0) = \frac{\mu_V(0)}{2\pi} = \sqrt{n}$$

$$\beta_V(0) = \frac{\rho}{\sqrt{n}},\ \alpha_V(0) = 0,\ \gamma_V(0) = \frac{\sqrt{n}}{\rho}$$

Momentum dispersion is

$$\eta(s) = \frac{2\rho}{1-n},\ \eta'(s) = 0$$

Our case is n=5E-3,
and then, parameters are :

$$\nu_H = \sqrt{1-n} \approx 1,\quad \beta_H \approx \rho = 33.3cm$$

$$\nu_v = \sqrt{n} \sim 5E-3,\ \beta_V \approx 1.83\times10^2\rho = 61m,$$

$$\eta \approx 0.66662\text{m}$$

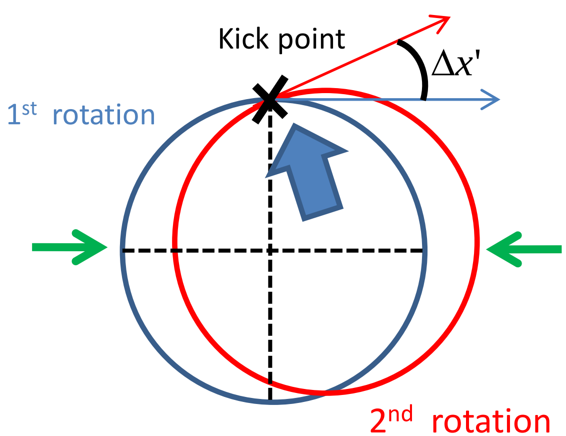
Error field estimation (Horizontal)

Due to n=3E-5 ≅0, M_H is expressed as

$$M_H \cong \begin{pmatrix} \cos\frac{s}{\rho} & \rho\sin\frac{s}{\rho} \\ -\frac{1}{\rho}\sin\frac{s}{\rho} & \cos\frac{s}{\rho} \end{pmatrix}$$

Assuming that beam suffers horizontal kick Δx' per turn: $\Delta x' \equiv \frac{\Delta Bl}{B\rho}$

Because of a matrix for an one turn $\approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ beam is kicked repeatedly at the ***same*** point, but the angle is changed by Δx'.



Difference between the 1st and 2nd rot. orbits is maximized at ¼ and ¾ rot. points by this kick. And its value is ρΔx'.

$$\text{Matrix} = \begin{pmatrix} 0 & \rho \\ -\frac{1}{\rho} & 0 \end{pmatrix}$$

After m rotations, orbits are moved by

$$x_m = \rho m(\Delta x')$$

If we request x_m <5mm for m=7000,

$$\Delta x' \equiv \frac{\Delta Bl}{B\rho} \leq 2.1\times10^{-6}$$

$$\because B\rho = 1[T\cdot m]$$

$$\Delta Bl \leq 2.1\times10^{-6}[T.m]$$

→200mGauss for l=10cm
→20mGauss for l=1cm

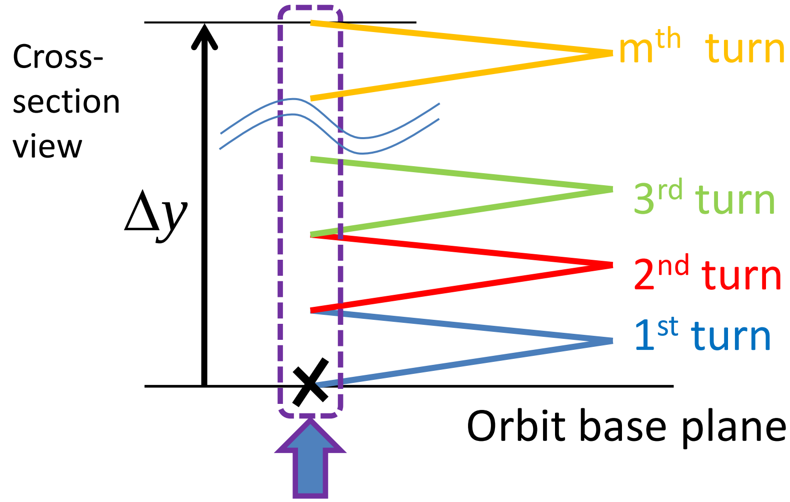
20mGauss/3T = 0.66E-6
→level of 1ppm in local!

Error field estimation (Vertical)

Due to n=3E-5 ≅0, M_V is expressed as

$$M_V \cong \begin{pmatrix} 1 & s \\ -\frac{n}{\rho^2}s & 1 \end{pmatrix}$$

This is an almost free space matrix and beam is kicked repeatedly at the ***same horizontal (or azimuthally)*** but slightly ***different vertical*** points. Vertical difference between the turns growths linearly (Δy).



$$\Delta y' \equiv \frac{\Delta B_R l}{B\rho}$$

$$\Delta y = 2\pi\rho\Delta y' \frac{m(m+1)}{2} - (2\pi)^3 n\rho\Delta y' \frac{(m-2)(m-1)}{2}$$

$$\approx 2\pi\rho\Delta y' \frac{m^2}{2}$$

$$1/\nu_V = 200$$

→Δy' changes its sign every ~100 turns!

If we request Δy<2cm for m=100,

$$\Delta y' \equiv \frac{\Delta B_R l}{B\rho} = \frac{\Delta y}{\pi\rho m^2} \le 1.9\times10^{-6}$$

$$\because B\rho = 1[T\cdot m]$$

$$\Delta B_R l \le 1.9\times10^{-6}[T.m]$$

→190mGauss for l=10cm vertically
→19mGauss for l=1cm
19mGauss/3T = 0.63E-6
→level of 1ppm in local!!

Acceptable error field is level of 1ppm in local for both horizontal and vertical