

# Design of a strong X-Y coupling beam transport line for J-PARC muon g-2/EDM experiment



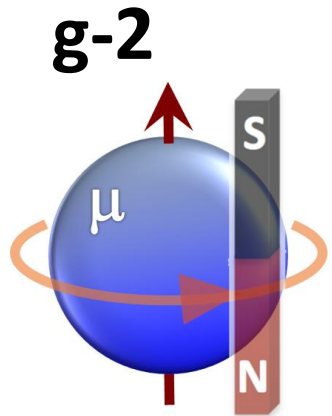
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Design of a beam transport line for a newly developed three-dimensional spiral injection scheme is discussed. This transport line is unique and one of key equipment for a new experiment at J-PARC, which measures a muon anomalous magnetic moment ( $g-2$ ) and electric dipole moment (EDM) to explore a new physics beyond the standard model. Very precise measurement on spin precession angular momentum of a muon in a high uniformity magnetic field will allow us to obtain these two fundamental physics values:  $g-2$  and EDM. We apply medical MRI type superconducting magnet technology to perform  $\pm 0.1$  ppm of high uniformity of three Tesla magnetic field. Relativistic energy of muon beam injection into such MRI sized magnetic field is the world first attempt. Because of axial symmetric field shape of a solenoid magnet, the beam phase-space should be strongly coupled in vertically (=solenoid axis) and radially (so called X-Y coupling), otherwise the stored beam diverges in vertically immediately. In order to avoid vertical dispersion of the stored beam, dedicated beam transport line is designed which realizes required X-Y coupling.

In this poster, we introduce (1) a transfer matrix of the entire beam transport line to meet required X-Y coupling, (2) arbitrarily angle rotating quadrupole magnets to realize X-Y coupling. We also discuss other challenges due to installation of the storage magnet (three Tesla superconducting magnet); (3) dedicated support system for arbitrary angle rotating quadrupoles on the 25-degree tilted transport line with respect to the horizontal plane.

This work was supported by JSPS KAKENHI Grant number JP19H00673.

# Physics goal: Explore the beyond standard model

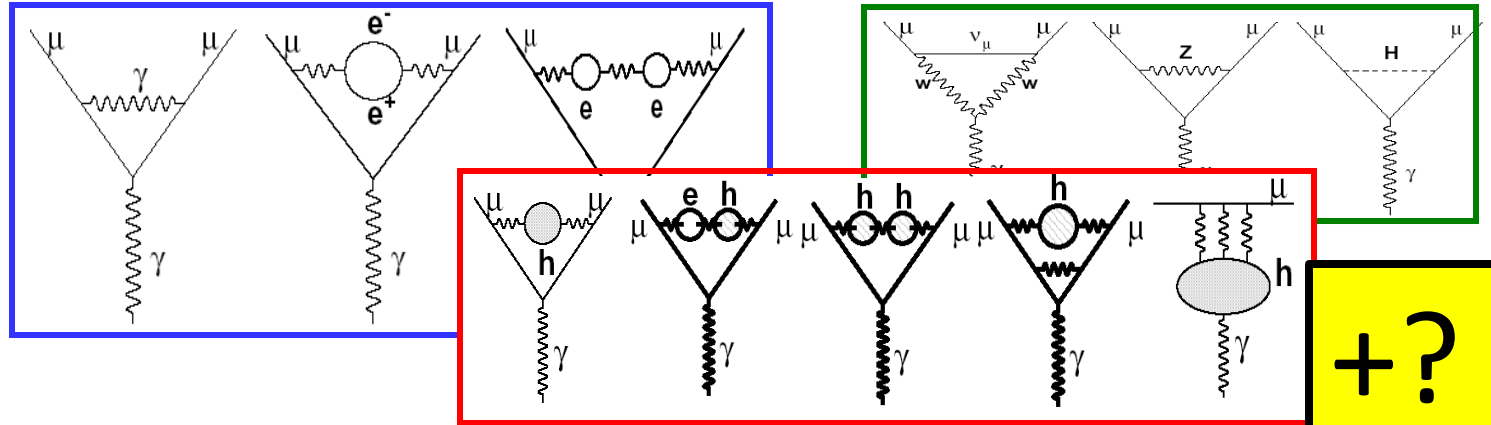


$$a_\mu = \frac{g - 2}{2} =$$

$$\vec{\mu}_s = \frac{g}{2} \left( \frac{e}{m_\mu} \right) \vec{s}$$

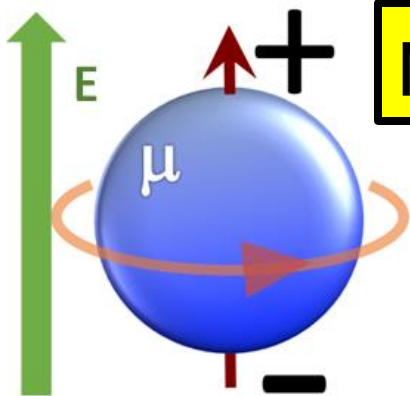
**g-2: 0.35ppm (exp.\*)**

**4.2σ discrepancy from the standard model**



(\*) combined two experiments;  
E989@FNAL, Phys. Rev. Lett. 126,  
141801(2021) & E821@BNL, Phys.  
Rev. D73 072003, 2006

**EDM**



**Non-zero observation = new physics**

Standard Model expects  $\sim 2 \times 10^{-38} \text{ e}\cdot\text{cm}$

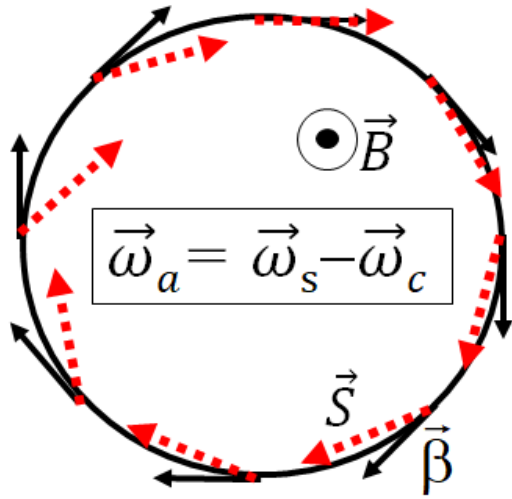
Upper limit (E821)  $< 1.9 \times 10^{-19} \text{ e}\cdot\text{cm}$  (90% CL)

E821@BNL, Phys. Rev. D 80, 052008, 2009

**We aim sensitivity of**

$$\sigma(d_\mu) < 1 \times 10^{-21} \text{ ecm}$$

# Muon spin precession probes new physics



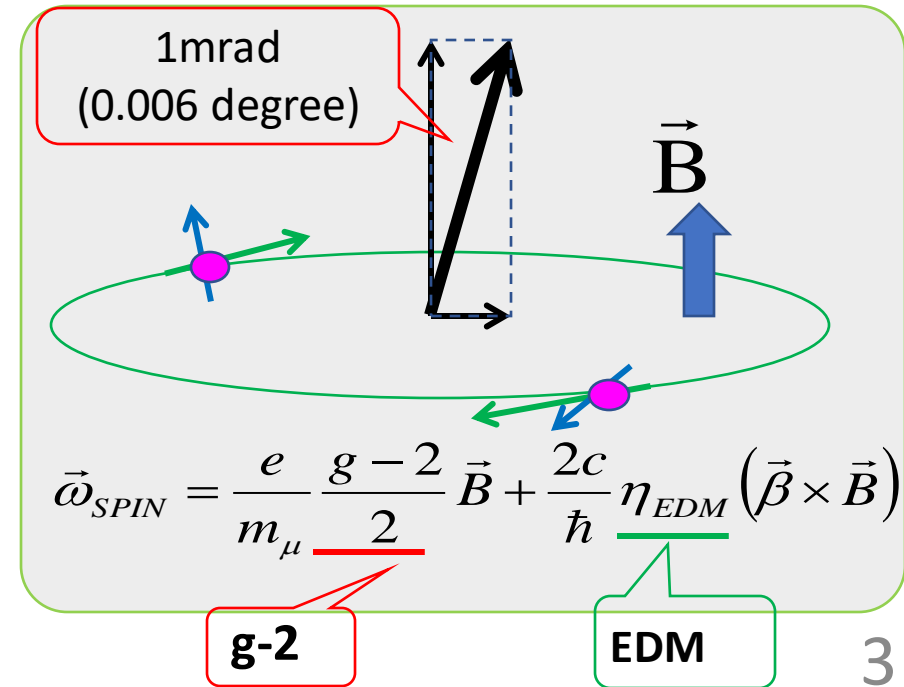
We measure  $\vec{\omega}_a = \text{Spin motion} - \text{cyclotron motion}$

$$\vec{\omega}_a = -\frac{q}{m} \left[ \underbrace{\left( \frac{g-2}{2} \right) \vec{B}}_{g-2} - \underbrace{\left( \frac{g-2}{2} - \frac{1}{\gamma^2-1} \right) \vec{\beta} \times \vec{E}}_{\text{EDM}} + \frac{\epsilon_{EDM}}{2} (\vec{\beta} \times \vec{B}) + \frac{\epsilon_{EDM}}{2} \vec{E} \right]$$

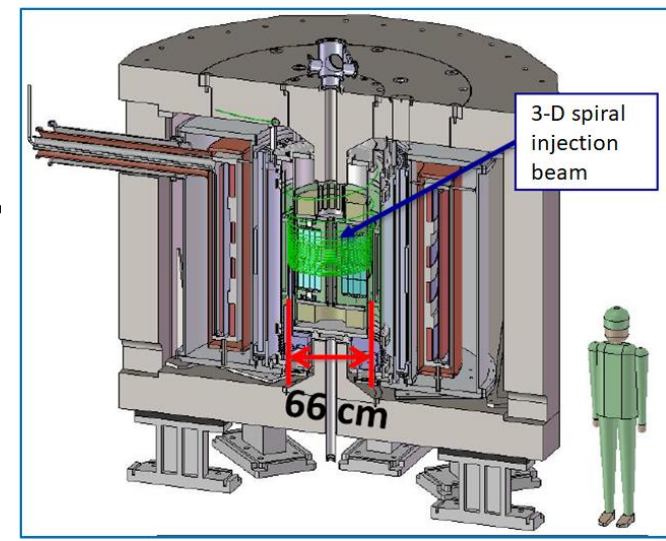
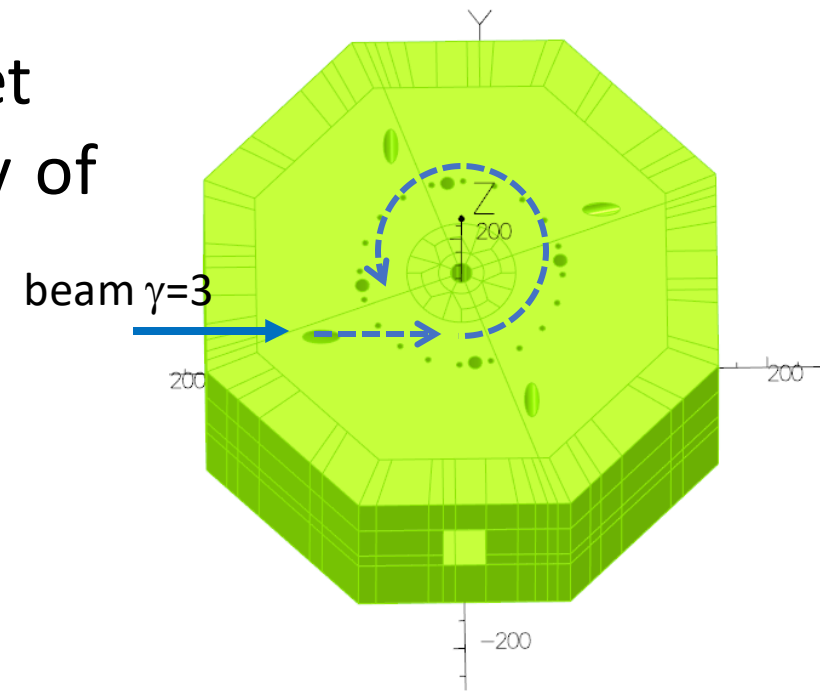
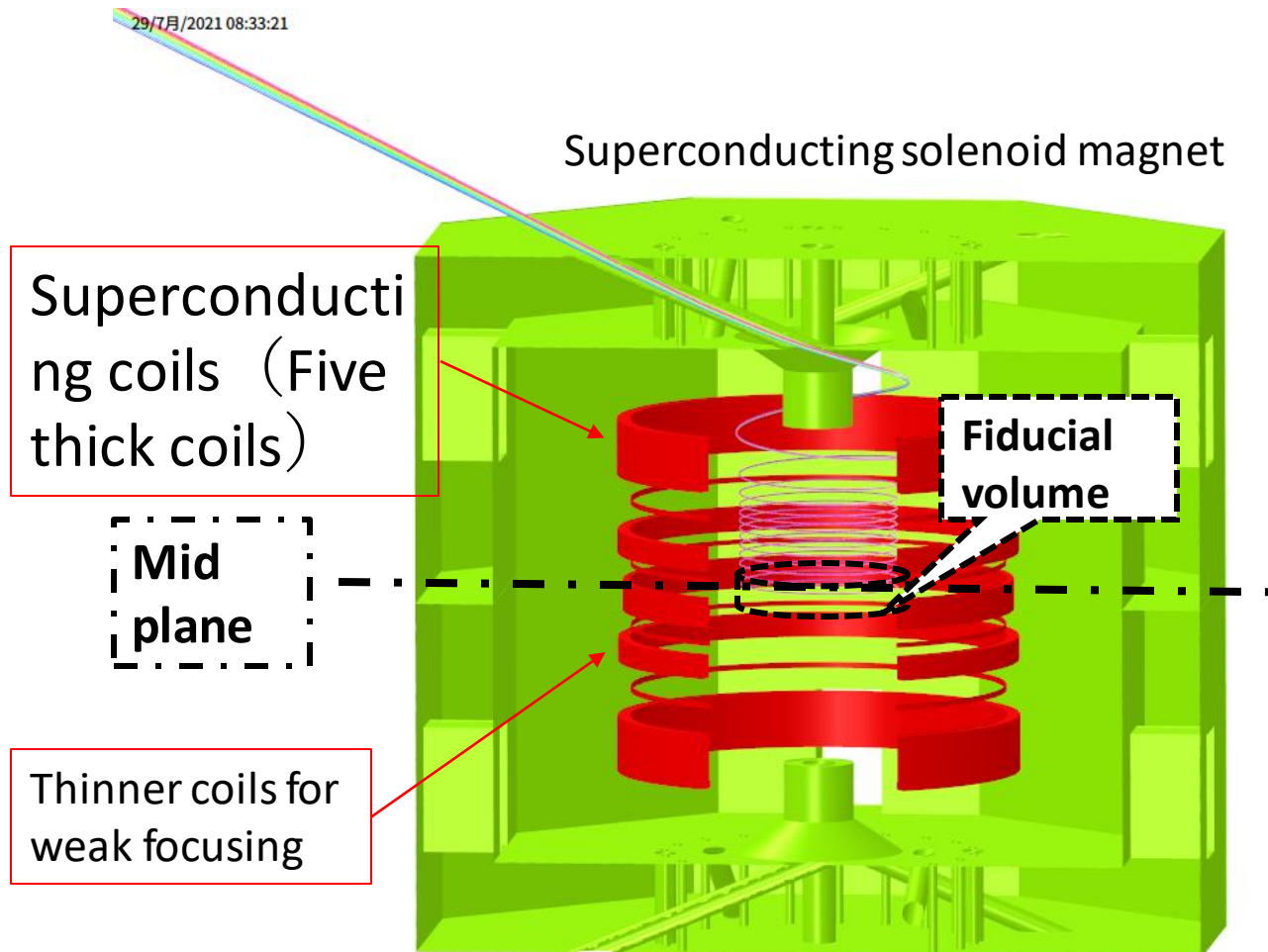
**EDM upper limit  $\sim 1e-19$  e.cm**

- ◆ Electric field  $\vec{E}=0$
- ◆ Store muon beam in the uniform magnetic field (<0.1ppm)
- ◆ Very precise control of the muon storage orbit
  - ◆ Angle between  $\vec{\omega}_a$  and magnetic field  $\vec{B}$  is estimated to be 1mrad assuming EDM upper limit from the previous experiment.
  - ◆ If we measure such angle with 0.01mrad precision, we perform very precise EDM measurement with 100 better sensitivity than previous exp.

We aim: **g-2 :0.45ppm (statistical uncertainty)**  
**(2021-E989 0.46ppm)**  
**EDM sensitivity:1.5e-21 e.cm**



We apply medical MRI type superconducting magnet technology to perform  $\pm 0.1$  ppm of high uniformity of three Tesla magnetic field



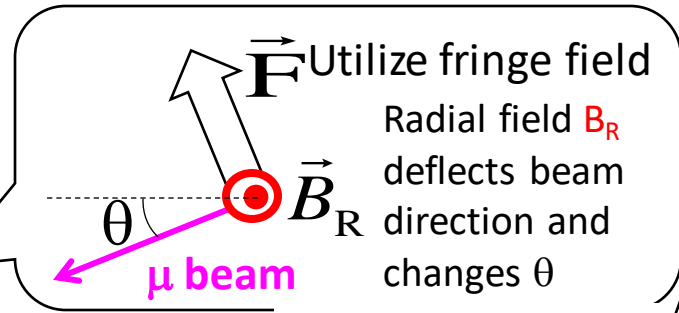
# How to inject the beam into MRI-sized compact storage magnet?

## Newly develop 3-D spiral injection

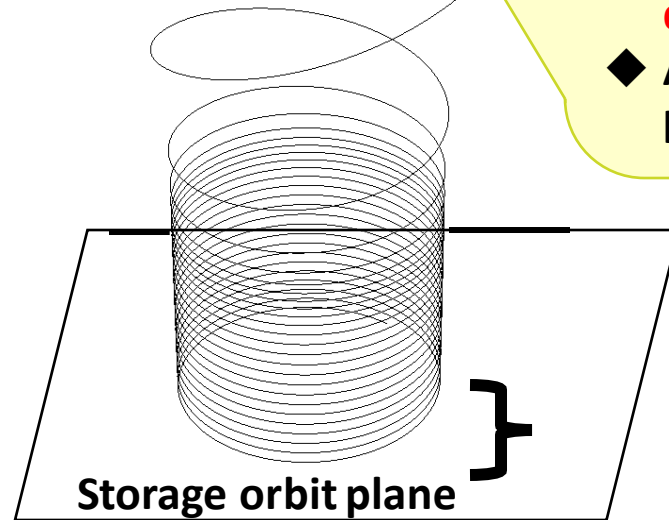
**New idea and unprecedented scheme!**

Beauties:

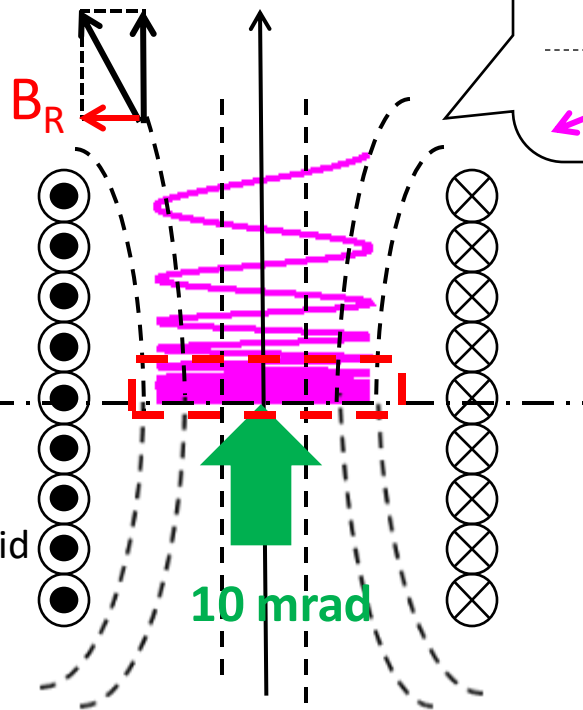
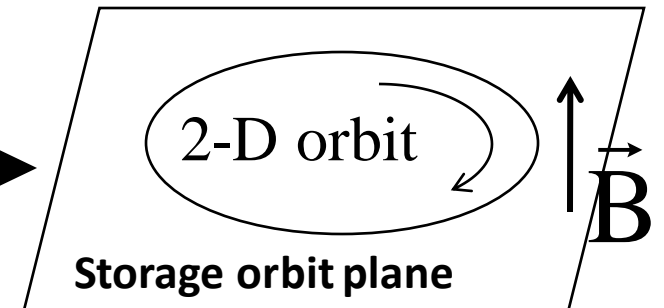
- ◆ Smooth connection between injection and storage sections without any sources of error field
- ◆ An unit of magnet does work for this method and decrease sources of error field
- ◆ Apply vertical kick during several turns (easier kicker requirement)



3-D spiral trajectory



Vertical kick



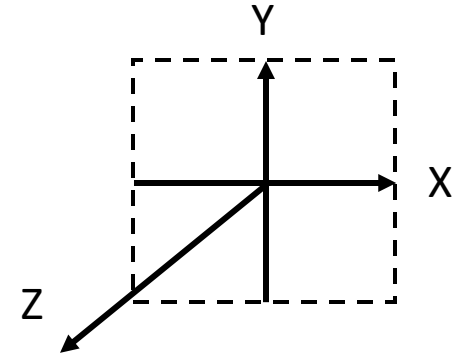
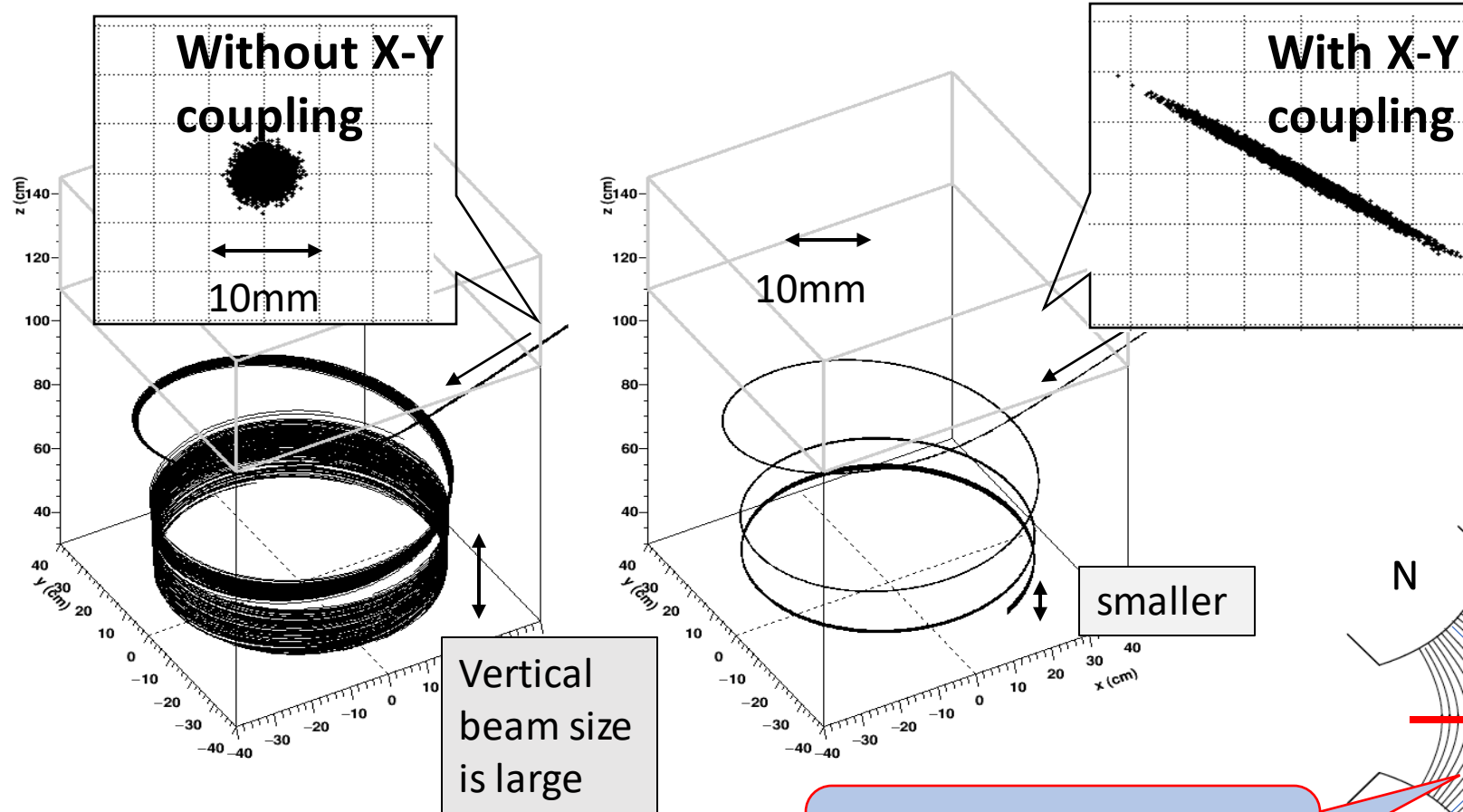
Dedicated control of muon beam phase space to inject beam into "axial symmetric" magnetic field, so called, X-Y coupling.

# What is "X-Y coupling"?

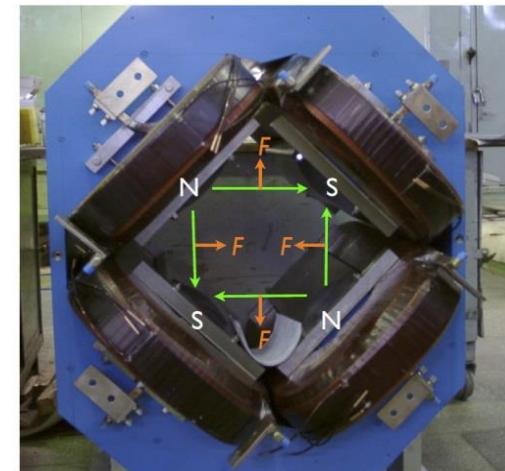
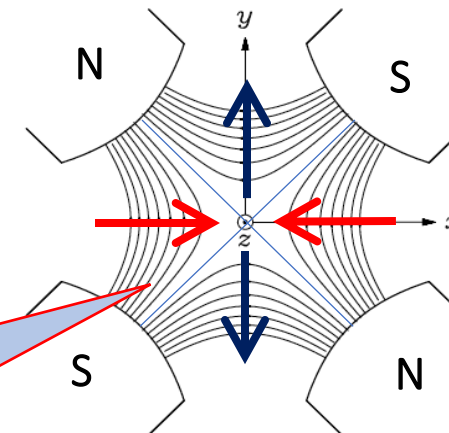
## A key technology for 3-D spiral injection scheme

Dedicated control of muon beam phase space to inject beam into "axial symmetric" magnetic field.

We need to control eight independent parameters of phase space (Twiss parameter:  $\alpha_x, \beta_x, \alpha_y, \beta_y$ , and  $r_1, r_2, r_3, r_4$  for X-Y coupling)

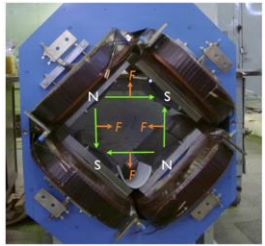


In general, beam motion in horizontal and vertical direction controlled independently by use of normal quadrupoles.

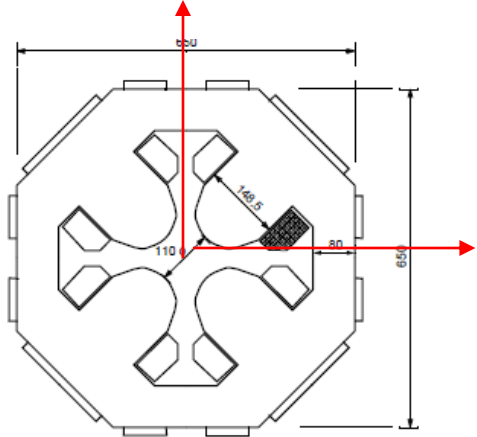
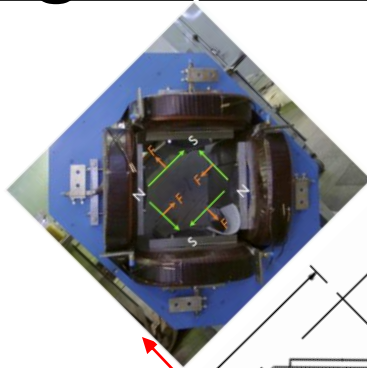


How do we apply X-Y coupling appropriately?

# Quadrupole magnet with appropriate rotation does help.

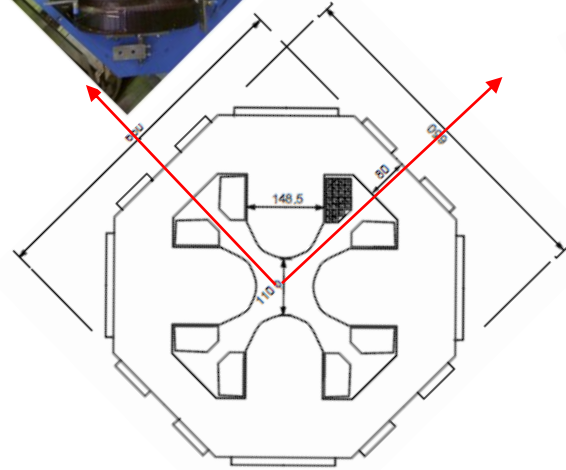


45 degrees



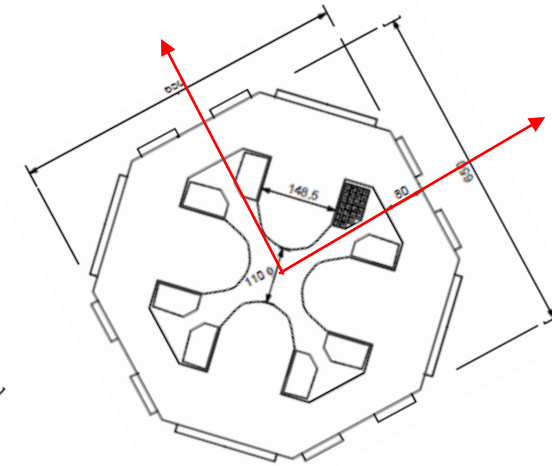
**Normal quadrupole**

Independent control beam motion horizontally (X-axis) and vertically (Y-axis). Beam goes along Z-axis.



**Skew quad**  
(45 degrees)

Sigma-matrix of beam phase space



**Rotating quadrupole**  
(arbitrary angle)

Useful to apply X-Y coupling

$$R(\varphi) = \begin{pmatrix} \cos \varphi & 0 & \sin \varphi & 0 \\ 0 & \cos \varphi & 0 & \sin \varphi \\ -\sin \varphi & 0 & \cos \varphi & 0 \\ 0 & -\sin \varphi & 0 & \cos \varphi \end{pmatrix}$$

Rotation matrix

$$\vec{x} = U\vec{X}$$

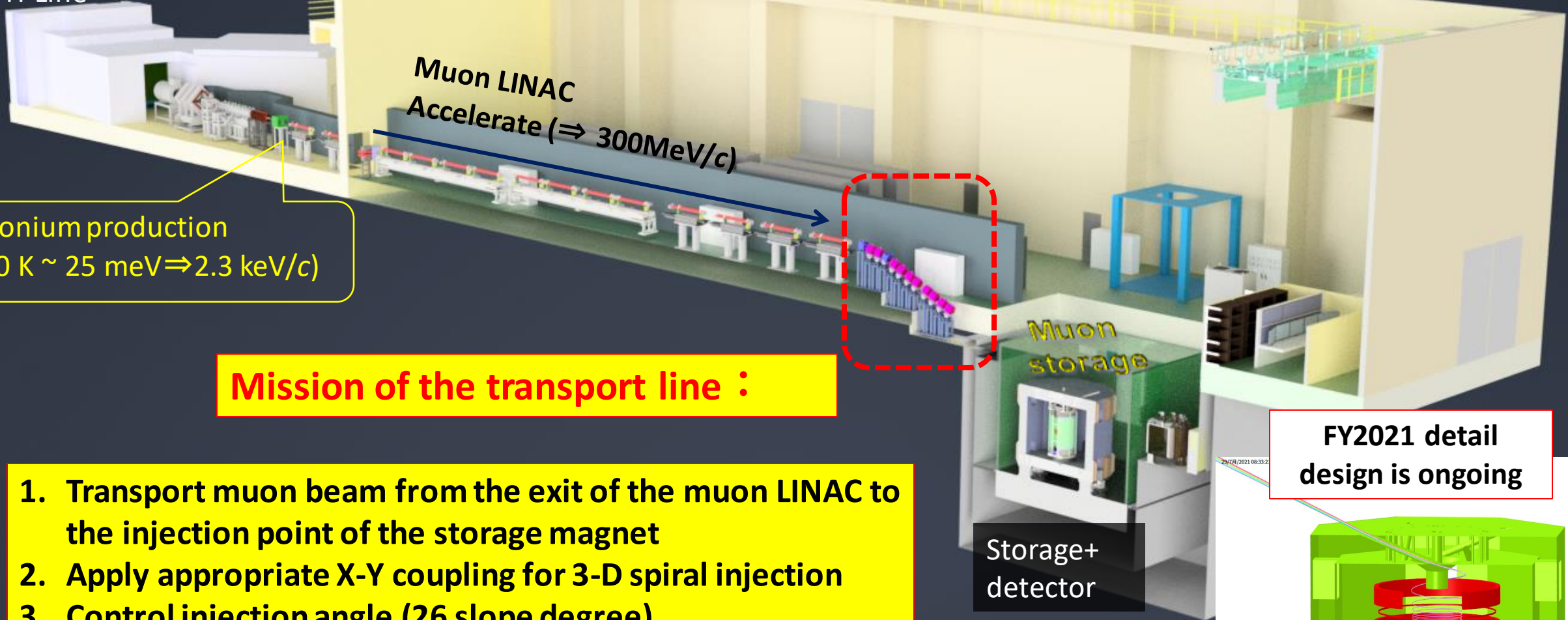
$$r = \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix}$$

$$U = \begin{bmatrix} \mu l & J_2^t r J_2 \\ r & \mu l \end{bmatrix} = \begin{bmatrix} \mu & 0 & -R_4 & R_2 \\ 0 & \mu & R_3 & R_1 \\ R_1 & R_2 & \mu & 0 \\ R_3 & R_4 & 0 & \mu \end{bmatrix}$$

$$\left( \begin{array}{cc|cc} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \hline \langle yx \rangle & \langle yx' \rangle & \langle y^2 \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'^2 \rangle \end{array} \right)$$

# Design of a strong X-Y coupling beam transport line

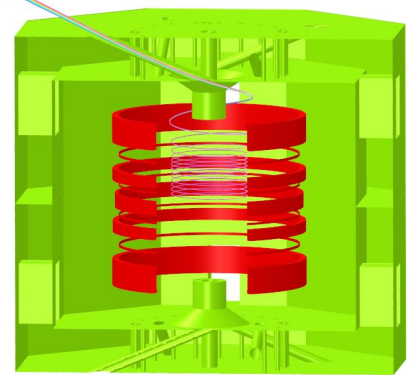
Beam line image at J-PARC  
MLF H-Line



## Mission of the transport line :

1. Transport muon beam from the exit of the muon LINAC to the injection point of the storage magnet
2. Apply appropriate X-Y coupling for 3-D spiral injection
3. Control injection angle (26 slope degree)

FY2021 detail design is ongoing

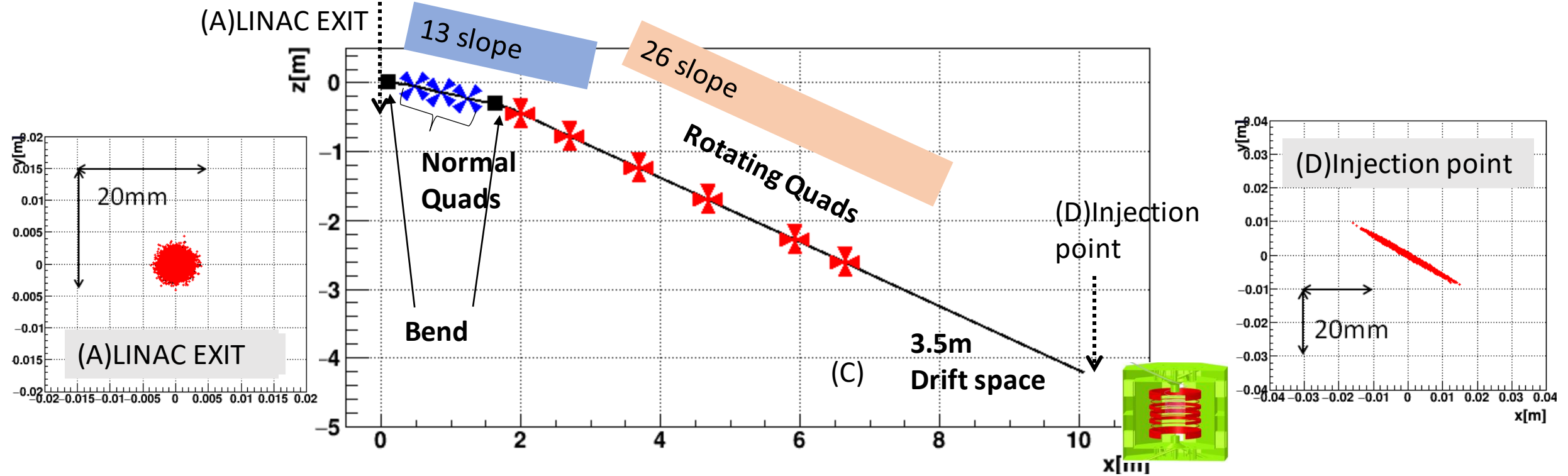




# Transfer matrix of transport line

## Seven rotating quadrupoles control X-Y coupling

(A) LINAC EXIT



Initial Sigma-matrix without X-Y coupling:  $\sigma_1$

$$\sigma_1 = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle y^2 \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'^2 \rangle \end{pmatrix}$$

where  $\langle xy \rangle, \langle x'y \rangle, \langle yx \rangle, \langle y'x \rangle$  are zero.

$$\sigma_2 = M \sigma_1$$

$$M = U_2^{-1} D U_1$$

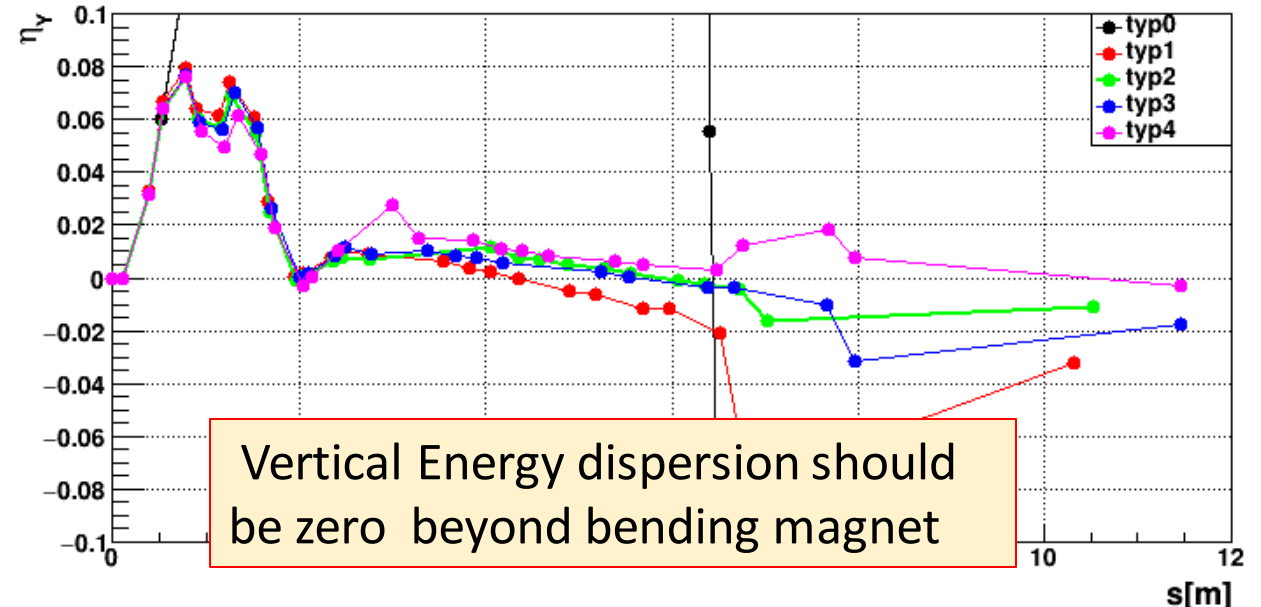
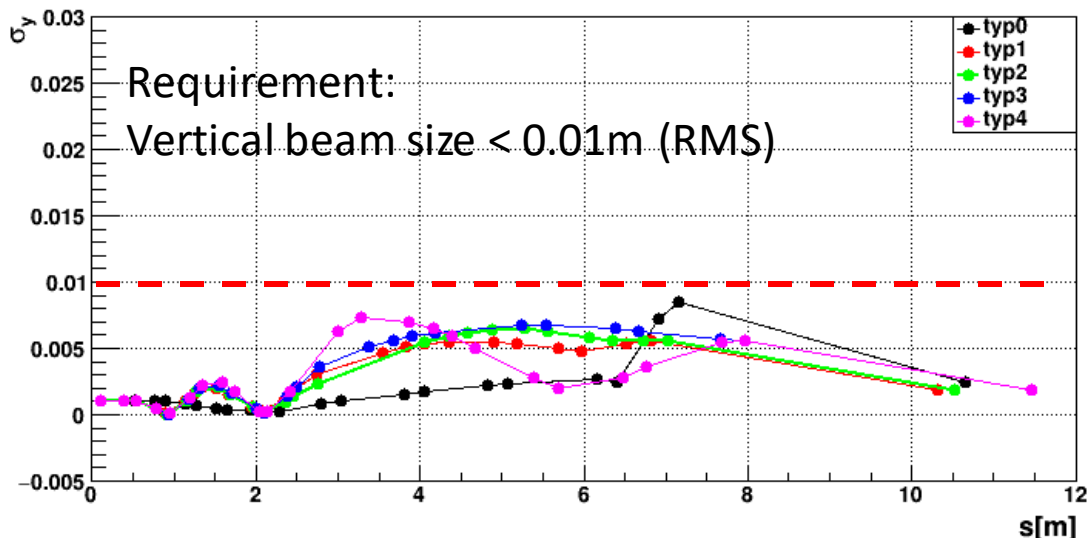
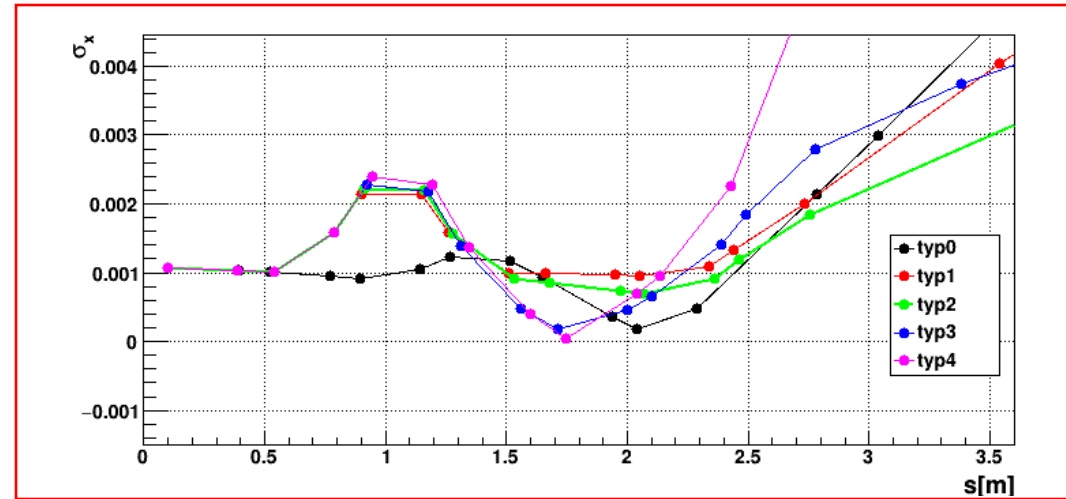
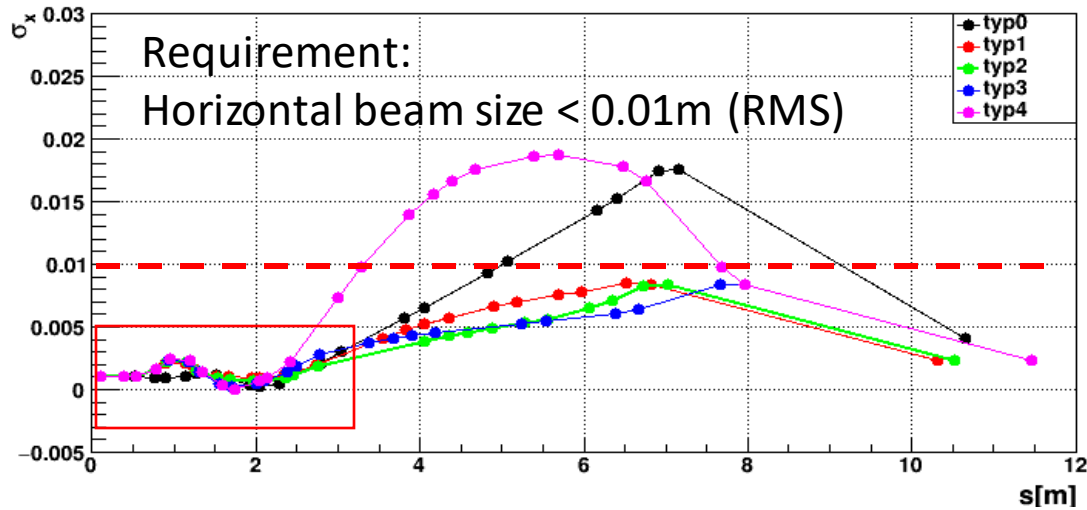
$$D = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$$

Sigma-matrix with appropriate X-Y coupling:  $\sigma_2$

$$\sigma_2 = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle y^2 \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'^2 \rangle \end{pmatrix}$$

where  $\langle xy \rangle, \langle x'y \rangle, \langle yx \rangle, \langle y'x \rangle$  are non-zero.

Five candidates of transfer matrix M. typ1,2 and 3 are acceptable.  
 Small  $\eta_y$  and tuning  $\sigma_x$  ( $\beta_x$ ) can keep beam size smaller



# Typ-2 is the best candidate

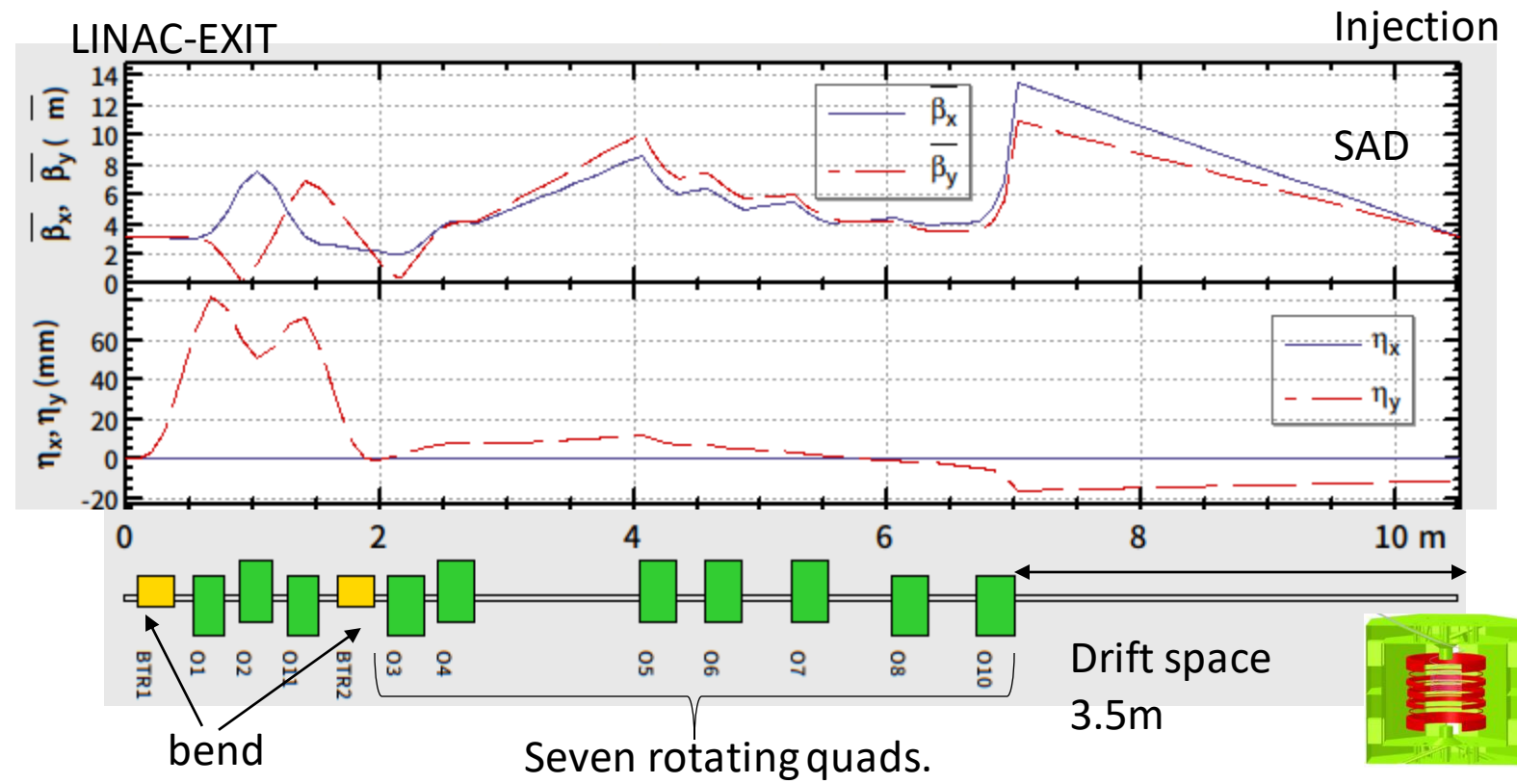
	Bore radius	K[T/m]	AT
Q1	0.01	-18.1	-720
Q2	0.01	17.4	691
Q11	0.01	-17.6	-702

Effective length 0.25m

	Bore radius	K[T/m]	AT	Angle(deg)
Q3	0.01	-20.2	-805	-30.6
Q4	0.01	3.7	147.5	-51.3

Effective length 0.29m

We fabricate Q10 in this fiscal year.



	Bore radius	K[T/m]	AT	Angle (deg)
Q5	0.03	0.71	252.8	-61.6
Q6	0.03	0.64	229.6	-57.2
Q7	0.03	1.04	372.0	-61.1
Q8	0.03	-0.64	-230.8	-36.8
Q10	0.03	-2.31	-828.3	-58.2

# Summary and next

- Preparation for new muon g-2/EDM experiment at J-PARC is ongoing.
- Muon g-2 and EDM probe new physics beyond the Standard Model.
- Discrepancy between experiment and theory  $> 4.2\sigma$ .

- Design work for the transport line (LINAC EXIT  $\sim$  injection point) is ongoing.
- X-Y coupling for the injection beam is important for 3-D spiral injection scheme.
- Set of rotating quadrupole magnets control appropriate X-Y coupling.
- Beam injection point is below 4m with respect to the beam line height of LINAC, and transport line need to treat 26 degrees slope.
- Mechanical design work for quads' base (right pict.) is ongoing.
- We will have a muon beam  $\sim$ FY2026.

We fabricate Q10 and it's base-stand in this FY2021.

Image of dedicated support system for arbitrary angle rotating quadrupoles.

