# Bend Matrix study

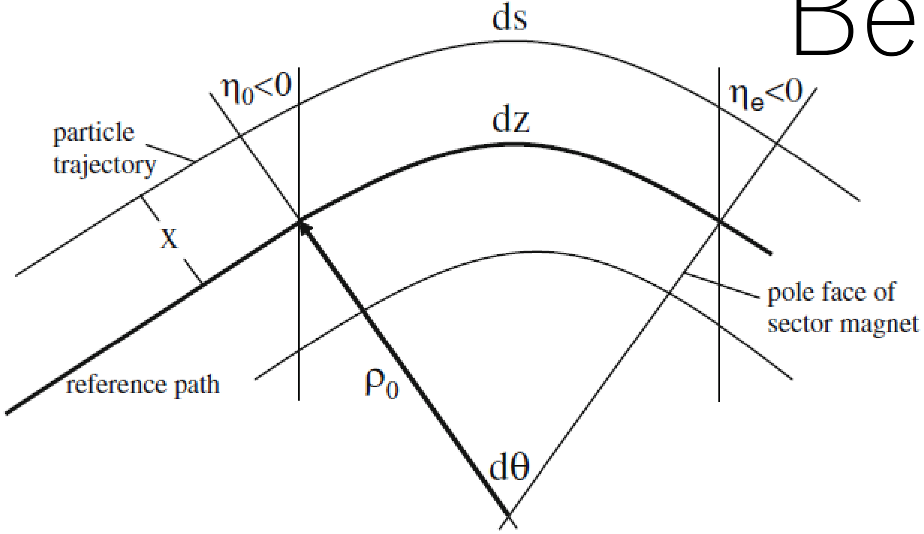
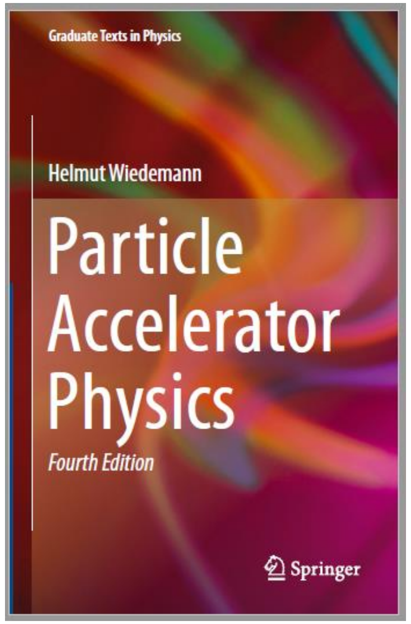



**Fig. 7.8** Focusing in a sector magnet, where $\eta_0 = \eta_e = 0$




The transformation matrix for a pure sector magnet of length $\ell$ and bending angle $\theta = \kappa_0 \ell$ in the deflecting plane becomes from (7.38)

上記のようなセクター磁石（入りと出の角度が磁極と垂直）で、かつ漏れ磁場効果がない理想的な場合の転送行列

ベンド面の転送行列（フォーカス）

$$\mathcal{M}_{s,\rho}(\ell|0) = \begin{pmatrix} \cos\theta & \rho_0 \sin\theta \\ -\kappa_0 \sin\Theta & \cos\Theta \end{pmatrix}. \quad (7.40)$$

θとΘが間違ってるけど。

If we also let $\kappa_0 \to 0$ we arrive at the transformation matrix of a sector magnet in the nondeflecting plane

ベンド面と垂直面の転送行列は自由空間

$$\mathcal{M}_{s,0}(\ell|0) = \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix}, \quad (7.41)$$

which has the form of a drift space. A pure dipole sector magnet therefore behaves in the non-deflecting plane just like a drift space of length $\ell$. Note that $\ell$ is the arc length of the magnet while the engineering magnet length might be given as the straight length between entry and exit point.

# 実際は、セクター磁石でも、矩形磁石でも漏れ磁場効果は無視できない。

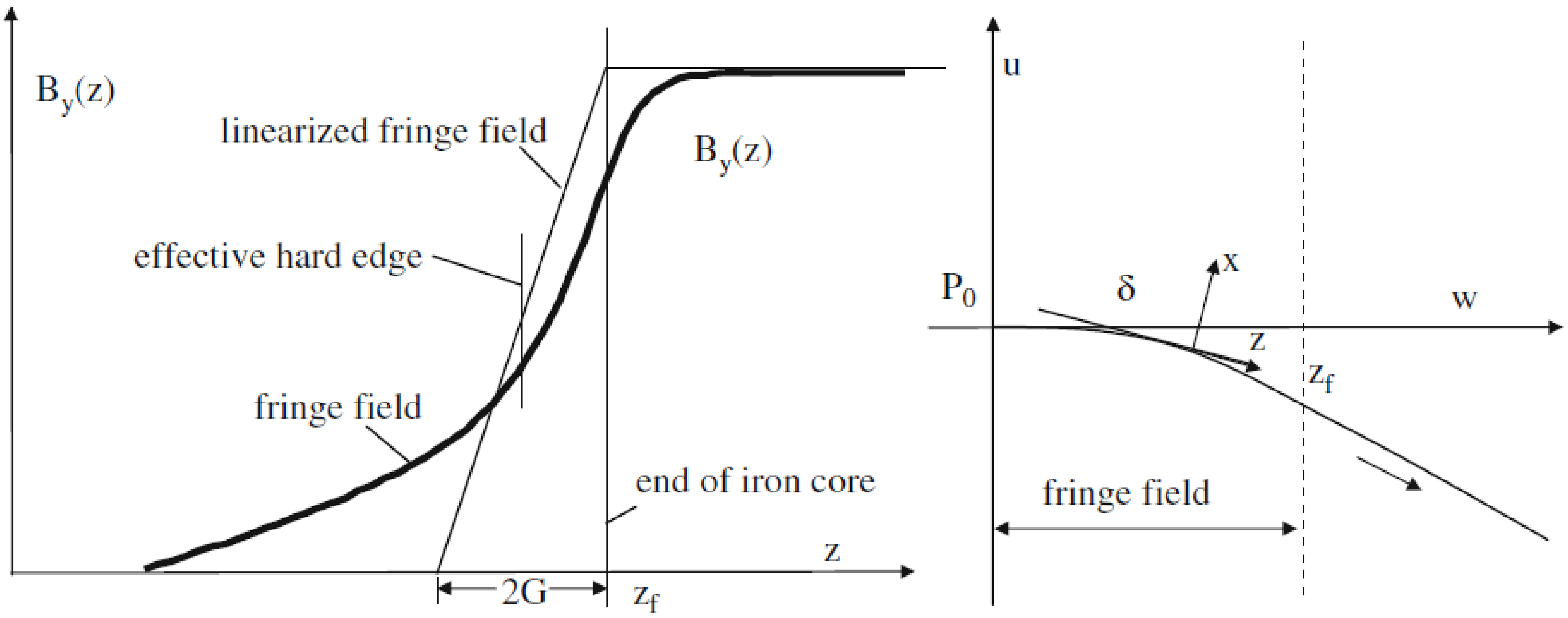



**Fig. 7.9** End field profile in a dipole magnet and fringe field focusing

depends on the particular field profile. We may approximate the fringe field by a linear fit over a distance approximately equal to the pole gap $2G$ which is a good approximation for most real dipole magnets. We neglect the nonlinear part of the

# ベンド面に関しては、フリンジ磁場もメインの磁場に盛り込んだ転送行列の形にできる。

$$\frac{1}{f_x} = \int_0^{z_f} (\kappa' \tan\delta + \kappa^2)\, d\bar{z}\,, \tag{7.51}$$

where we have set $\kappa(z) = (e/p_0)\,F(z)$. For small deflection angles $\delta$ in the fringe field $\tan\delta \approx \delta = \int_0^{z_f} \kappa\, d\bar{z}$ and after integration of (7.48) by parts through the full fringe field we get the focal length while neglecting higher order terms in $\delta_f$

where $\kappa_0 = 1/\rho_0$ is the curvature in the central part of the magnet and $\delta_f$ is the total deflection angle in the fringe field region.

This result does not deviate from that of the hard edge model, where for a small deflection angle $\delta$ we have from (7.40) $1/f_x \approx \kappa_0\,\delta$ agreeing with (7.52). We obtain therefore the convenient result that in the deflecting plane of a sector magnet there is no need to correct the focusing because of the finite extend of the fringe field.

$$\mathcal{M}_{s,\rho}(\ell|0) = \begin{pmatrix} \cos\theta & \rho_0 \sin\theta \\ -\kappa_0 \sin\Theta & \cos\Theta \end{pmatrix}. \tag{7.40}$$

θとΘが間違ってるけど。

# ベンドと垂直面に関しては、フリンジ磁場の影響を考慮せねばならない。

With these results we may now derive a corrected transformation matrix for a sector magnet by multiplying the hard edge matrix (7.41)on either side with thin length fringe field focusing

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_y} & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_y} & 1 \end{pmatrix} \tag{7.57}$$

フリンジ(出口)　　フリンジ(入り口)

and get with (7.55) and $\theta = \ell/\rho_0$ for the transformation matrix in the vertical, non-deflecting plane of a sector magnet instead of (7.41)

$$\mathcal{M}_{s,0}(\ell\,|\,0) = \begin{pmatrix} 1 + \frac{1}{3}\theta\,\delta_f & \ell \\ \frac{2}{3}\frac{\delta_f}{\rho_0} - \frac{1}{9}\frac{\delta_f^2}{\rho_0^2}\ell & 1 + \frac{1}{3}\theta\,\delta_f \end{pmatrix}. \tag{7.58}$$

$$\delta_f = \kappa_0\,G\,. \tag{7.56}$$

# 矩形磁石の場合（我々）　入りと出口の角度αの定義（教科書に依って異なるが、SADはこの定義）

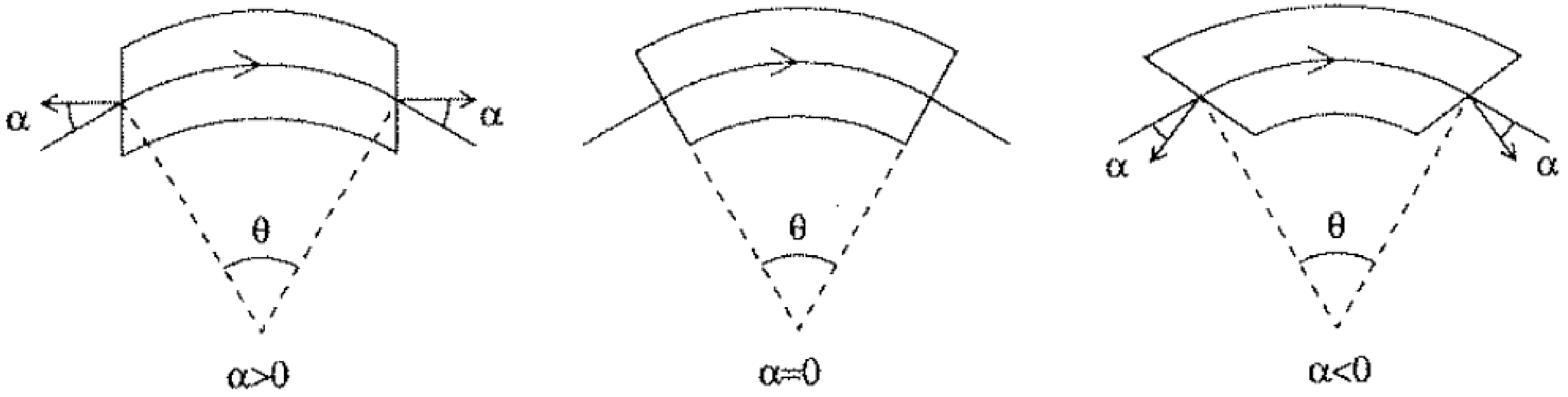



Figure. 4.3 The three most common situations regarding the angle $\alpha$ formed by the beam direction and the normal $\vec{n}$ to the magnet yokes.

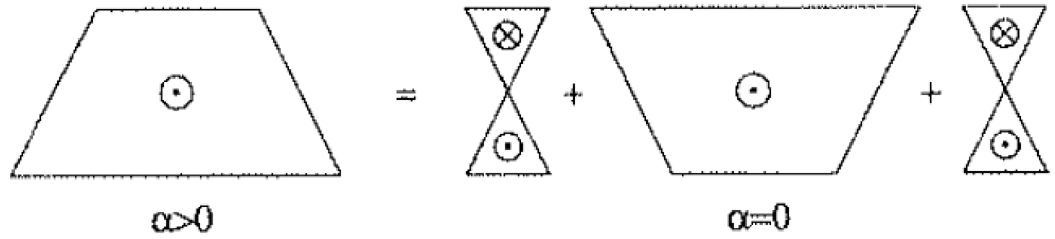



Figure. 4.4 A magnet with positively slanted ends, $\alpha > 0$, can be considered as the combination of a normal bending magnet and two magnetic wedges.

The horizontal displacement for a thin wedge is negligible, $\Delta x \simeq 0$, and the effect of the kick may be represented by the matrix

**ベンド面の転送行列（矩形の場合）**

$$\mathbf{M}_{\mathrm{W}} = \begin{pmatrix} 1 & 0 \\ \frac{\tan\alpha}{\rho} & 1 \end{pmatrix}, \tag{4.36}$$

or

$$\mathbf{M}_{\mathrm{W}} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{\tan\alpha}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.37}$$

if dispersion is taken into account.

It is trivial to write down the matrix characterizing the sum of magnetic elements of Fig. 4.4;

フリンジ(出口)　フリンジ(入り口)

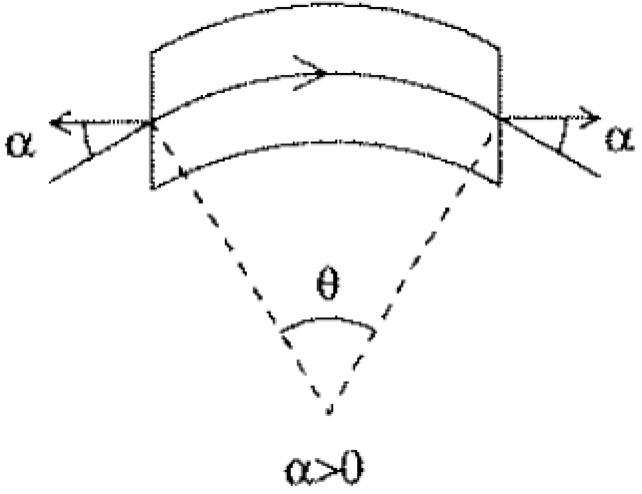
$$\mathbf{M}_{\mathrm{H}} = \mathbf{M}_{\mathrm{W}}\mathbf{M}_{\mathrm{H}}(\theta)\mathbf{M}_{\mathrm{W}}, \tag{4.38}$$

違う教科書だけど、この行列。
θとΘが間違ってるけど。

$$\mathcal{M}_{\mathrm{s},\rho}(\ell|0) = \begin{pmatrix} \cos\theta & \rho_0\sin\theta \\ -\kappa_0\sin\Theta & \cos\Theta \end{pmatrix}. \tag{7.40}$$

# もし、入りと出のαが同じなら

**ベンド面の転送行列（矩形の場合）**



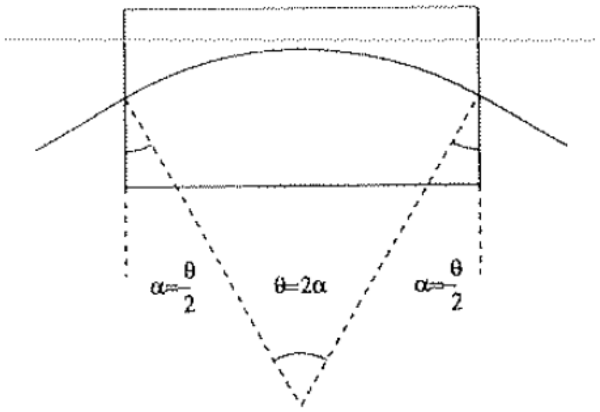



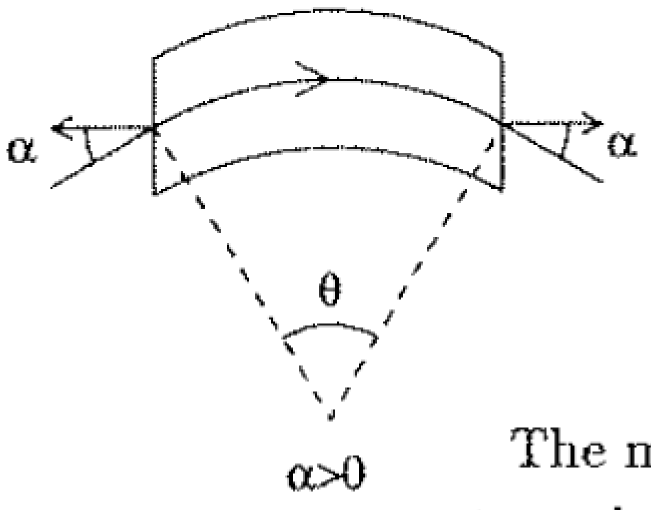

Figure. 4.7 A rectangular magnet.

The most common and interesting case occurs when the magnet's poles have a rectangular plan, as shown in Fig. 4.7. Then for $\alpha = \theta/2$,

$$\mathbf{M}_{\mathrm{H}} = \begin{pmatrix} 1 & \rho\sin\theta & \rho(1-\cos\theta) \\ 0 & 1 & 2\tan\frac{\theta}{2} \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.52}$$

# ベンド面と垂直面の転送行列（矩形の場合）

フリンジ(出口)　　フリンジ(入り口)




$$\mathbf{M}_V = \begin{pmatrix} 1 & 0 \\ -\frac{\tan\alpha}{\rho} & 1 \end{pmatrix} \begin{pmatrix} 1 & \rho\theta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{\tan\alpha}{\rho} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\theta\tan\alpha & \rho\theta \\ -\frac{\tan\alpha}{\rho}(2-\theta\tan\alpha) & 1-\theta\tan\alpha \end{pmatrix}. \qquad (4.51)$$

The most common and interesting case occurs when the magnet's poles have a rectangular plan, as shown in Fig. 4.7. Then for $\alpha = \theta/2$,

$$\mathbf{M}_V = \begin{pmatrix} 1-\theta\tan\frac{\theta}{2} & \rho\theta \\ -\frac{1}{\rho}\tan\frac{\theta}{2}\left(2-\theta\tan\frac{\theta}{2}\right) & 1-\theta\tan\frac{\theta}{2} \end{pmatrix}. \qquad (4.53)$$

For $\theta \ll 1$, keeping quadratic terms in $\theta$,

$$\mathbf{M}_V \simeq \begin{pmatrix} 1-\frac{\theta^2}{2} & \rho\theta \\ -\frac{\theta}{\rho} & 1-\frac{\theta^2}{2} \end{pmatrix} \simeq \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\frac{\sin\theta}{\rho} & \cos\theta \end{pmatrix}, \qquad (4.55)$$

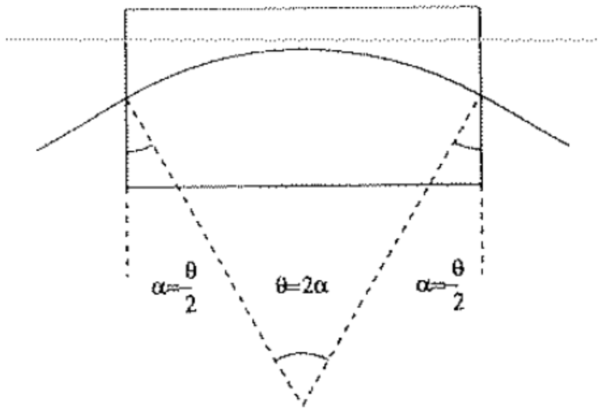



Figure. 4.7 A rectangular magnet.

ベンド面の転送行列と同じ形になる。

For $\theta \ll 1$, keeping quadratic terms in $\theta$, the horizontal matrix reduces to

$$\mathbf{M}_{\mathrm{H}} = \begin{pmatrix} 1 & \rho\theta & \frac{1}{2}\rho\theta^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.54}$$

and the vertical matrix becomes

$$\mathbf{M}_{\mathrm{V}} \simeq \begin{pmatrix} 1 - \frac{\theta^2}{2} & \rho\theta \\ -\frac{\theta}{\rho} & 1 - \frac{\theta^2}{2} \end{pmatrix} \simeq \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\frac{\sin\theta}{\rho} & \cos\theta \end{pmatrix}, \tag{4.55}$$

which means that what is lost in horizontal focusing is gained in the vertical one. Nevertheless, for $\theta \ll 1$, the $2 \times 2$ nondispersive part of the horizontal matrix is almost equal to the vertical matrix:

$$\mathbf{M}_{\mathrm{H}} \simeq \mathbf{M}_{\mathrm{V}} \simeq \begin{pmatrix} 1 & \rho\theta \\ 0 & 1 \end{pmatrix}, \tag{4.56}$$

and the vertical and horizontal are almost the same as a simple drift of length $\rho\theta$.